IMAGE COMPRESSIVE-SENSING RECOVERY USING STRUCTURED LAPLACIAN SPARSITY IN DCT DOMAIN AND MULTI-HYPOTHESIS PREDICTION

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ABSTRACT

In compressive sensing (CS), the seeking of a fair domain is of essentially significance to achieve a high enough degree of signal sparsity. Most methods in the literature, however, use a fixed transform domain or prior information that cannot exhibit enough sparsity for various images. Superiorly, we propose an algorithm to explore the structured Laplacian sparsity of DCT coefficients, which can adapt to the nonstationarity of natural images. Better sparsity is achieved by utilizing the nonlocal similarity of natural images and constructing structured image patch groups. Meanwhile, multiple hypotheses for each pixel could be obtained owing to the overlapping of the structured groups and similar patches. Additionally, for solving the optimization problem formulated from the techniques above, we design an efficient iterative method based on split Bregman iteration (SBI) algorithm. Experimental results demonstrate that the proposed algorithm outperforms the other state-of-the-art methods in both objective and subjective recovery quality.

Index Terms—Compressive sensing, image recovery, structured Laplacian sparsity, multi-hypothesis

1. INTRODUCTION

Compressive Sensing (CS) has attracted tremendous interest in recent years, which provides the possibility of recovering a signal at sub-Nyquist rate [1]-[5]. It declares that a signal with sparse representations under some domain can be reconstructed with high probability from very few measurements, which are obtained via linearly projecting the original signal onto a random basis. CS theory depicts a new paradigm for signal acquisition, which conducts sampling and compression at the same time, rather than sequentially performing these two steps as the traditional methodology does. CS-based compression has an asymmetric design: simple encoder and complex decoder, which is quite conductive to some image processing applications where the data acquisition devices have to be simple (e.g. inexpensive resource-deprived sensors), or oversampling may harm the object being captured (e.g. X-ray imaging) [6].

It is believed in CS theory that how much sparsity a signal exhibits is very crucial in how well the signal can be recovered. That is to say, the sparser the signal is in the specified domain, the higher recovery quality could be yielded. In different domains, the image signal exhibits different degree of sparsity, thus seeking a desirable domain becomes one of the main challenges with which the CS recovery research is confronted. Fixed domains or a set of fixed bases (e.g. discrete cosine transform, wavelet and contourlet, gradient domain) [7], [8] are lately explored in a lot of CS recovery methods. Although using these domains is intuitively comprehensible, the restoration results are far from satisfactory due to their insufficiency in individually or adaptively representing the signals.

To deal with this problem, it is suggested incorporating additional prior knowledge (statistical dependencies, structure, etc.) into the CS recovery framework in the current literature. Typical works are Gaussian scale mixtures (GSM) models [8], tree-structured wavelet [9] and tree-structured DCT (TSDCT). Additionally, in [10], a projection-driven CS recovery coupled with block-based random image sampling is presented, aiming to encourage sparsity in the domain of directional transforms. Chen et al. [11] exploited multihypothesis predictions to generate residuals in the domain of the CS random projections, where the residuals are distinctly more compressible than the original signal. In [12], Zhang et al. proposes a scheme called structural group sparse representation (SGSR), which employs the nonlocal selfsimilarity of images by constructing groups of similar patches with individual dictionaries learned for each group. However, the inherent correlations among patches within each group are not considered and the sparsity lying in the group is not fully developed.

In this paper, we develop an image CS recovery algorithm using the structural Laplacian sparsity in discrete cosine transform (DCT) domain associated with the multihypothesis predictions. We have three major contributions. First, we achieve prominent Laplacian sparsity of DCT coefficients by structuring the image patch groups according

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to the nonlocal similarity of natural images. The structural Laplacian sparsity could be casted into the CS framework and an ℓ_1 optimization problem is formulated. Second, multiple hypotheses for each pixel are generated owing to the overlapped group construction and the superposition of similar patches. Hereby, we leverage the multi-hypothesis (MH) theory and are able to obtain a more accurate restoration result. Finally, for solving the optimization problem formulated from the techniques above, we design an efficient iterative method based on split Bregman iteration (SBI) algorithm.

The remainder of this paper is organized as follows. Section 2 is an overview of related background, providing a basic idea of the compressive sensing theory. Section 3 gives a detailed description of the proposed structured Laplacian sparsity based image recovery algorithm and the solution to the formulated optimization problem. Simulation results are provided in Section 4 and conclusions are drawn in Section 5.

2. BACKGROUND

The compressive sensing states that the signal can be recovered from very few samples, if it is sufficiently sparse in some domain Ψ . Concretely, suppose x is the original signal of length N, y is the measurement of length $M \ll N$ after sampling and the two of them satisfy y = Ax, in which A is the random projection matrix. If the coefficients of $\alpha = \Psi^T x$ are mostly zeroes or very close to zeroes, where Ψ is a transform basis, x could be restored through the following optimization problem

$$\min_{\mathbf{x}} \left\| \mathbf{\Psi}^T \mathbf{x} \right\|_p, \quad \text{s.t. } \mathbf{y} = \mathbf{A}\mathbf{x}. \tag{1}$$

In this formula, the subscript *p* is usually set to 0 or 1 to characterize the sparsity of the vector $\Psi^T x$. When *p* is 0, the objective becomes an ℓ_0 norm $\|*\|_0$, which counts the number of non-zero coefficients of *; when *p* is 1, the objective is an ℓ_1 norm $\|*\|_1$, which adds up the absolute values of all coefficients of *.

In practice, Eq. (1) is usually converted into an unconstrained problem by introducing a penalty parameter λ

$$\min_{\mathbf{x}} \frac{1}{2} \| \boldsymbol{y} - \boldsymbol{A} \boldsymbol{x} \|_{2}^{2} + \lambda \| \boldsymbol{\Psi}^{T} \boldsymbol{x} \|_{p}, \qquad (2)$$

which can be regarded as a regularization-based framework for CS recovery. Hereinto, the first term is referred to as the data fidelity term and the second one is the regularization term.

The goal of CS research is mostly to find a decent domain Ψ in which the signal x exhibits adequate sparsity. Conventionally, Ψ is made to be a set of fixed transform bases, such as the DCT matrix or the wavelet transform, to characterize the image sparsity in these domains. A constant basis for all images or all image blocks, however, cannot describe the non-stationarity of natural images, resulting in its incapability to adaptively recover various image contents. Therefore, the intrinsic nature of images should be taken into

consideration so that better prior information could be exploited to fit into the formula defined by Eq. (2) in the place of the regularization term. The nonlocal similarity of images, which depicts the consistency of image patches in the neighboring regions, is verified to be an extraordinarily effective characteristic.

3. PROPOSED METHOD

As is widely known in the image processing area, the DCT coefficients of image blocks are proved to follow the Laplacian distribution [17], [18], which indicates the closeness of corresponding coefficients. Fig. 1 (a) depicts an example of the coefficient distribution in one frequency band.

Conventional CS methods based on this feature of images are mostly considering the statistic characteristics of all the transformed blocks for the entire image, i.e. Laplacian modelling in terms of the image. This cannot achieve an accurate model for different images and even for one image desirable sparsity cannot be obtained due to contents varying from regions to regions. Comparatively, we base our algorithm on the non-local similarity of images and construct groups containing patches similar in structure, as shown in Fig. 1 (b). Each patch is modeled with the Laplacian distribution specific to its group, which yields more accurate and robust sparse representation. Hereby, this characteristic is referred to as structured Laplacian sparsity.



Fig. 1. Structured Laplacian sparsity of image groups in DCT domain: (a) Laplacian distribution of DCT coefficients; (b) patches in a group exhibit more Laplacian sparsity due to their similarity

3.1. Image Nonlocal Similarity and Structured Group Construction



The squares of the same color indicate patches repetitively occurring in the nonlocal areas of the image, which demonstrates the nonlocal similarity characteristic.

Fig. 2. Example of the nonlocal self-similarity of images

Nonlocal similarity first proposed for image denoising [14] is a significant property of natural images. It characterizes the repetitiveness of the textures or structures embodied by natural images within the nonlocal area as shown in Fig. 2 and can be used for retaining the sharpness and edges effectually to maintain image nonlocal consistency [14]-[16]. We first present the construction of structured groups by exploiting the non-local similarity and the process is described as follows.

We partition the image \mathbf{x} with size of N into D overlapped patches and for each patch search for patches alike in its neighborhood to form a patch group. As illustrated in Fig. 3 (a), the red solid square represents a patch of size S^2 , which we denote as $\mathbf{x}_k, 1 \le k \le D$. In the surrounding area of the patch \mathbf{x}_k , we define a search window of size L^2 marked by the dashed blue square, within which the most similar C patches $\mathbf{x}_{k,i}, (1 \le i \le C)$ are selected to construct a group denoted by $\mathbf{G}_{\mathbf{x}_k} = {\mathbf{x}_{k,1}, \mathbf{x}_{k,2}, \dots, \mathbf{x}_{k,C}}$. In Fig. 3 (b), the red solid square is the central patch \mathbf{x}_k in Fig. 3 (a) and it along with all the blue solid squares, which signifies its similar patches, makes up the group $\mathbf{G}_{\mathbf{x}_k}$.

To formulate this process mathematically, without confusion the notation $\mathbf{x}_{k,i}$ is used to represent the vector containing all pixels in the patch as well as the patch itself. Specifically, we fetch all the pixels in each patch in raster scan order and place them in turn in a column vector denoted by $\mathbf{x}_{k,i}$. Accordingly, the notation $\mathbf{G}_{\mathbf{x}_k}$ is also defined as the matrix with all the column vector $\mathbf{x}_{k,i}$, $(1 \le i \le C)$ concatenated together, namely $\mathbf{G}_{\mathbf{x}_k} = [\mathbf{x}_{k,1}, \mathbf{x}_{k,2}, ..., \mathbf{x}_{k,C}]$. Which of these two concepts the notations are referring to will be specified in the context in the following sections.



Fig. 3. Illustration of image patch group construction

3.2. Structured Laplacian Sparsity Prior in DCT Domain

This subsection is elaborating the formulation of the structured Laplacian sparsity in each constructed group.

As stated in Section 3.1, to exploit the nonlocal similarity of natural images, we construct groups containing resembled patches, which are evidently supposed to better exhibit the Laplacian sparsity in the DCT domain. Here, we make Φ denote the DCT transform bases and define

$$\tilde{\boldsymbol{G}}_{\boldsymbol{x}_{k}} = \boldsymbol{\Phi}^{T} \boldsymbol{G}_{\boldsymbol{x}_{k}} = [\boldsymbol{\Phi}^{T} \boldsymbol{x}_{k,1}, \boldsymbol{\Phi}^{T} \boldsymbol{x}_{k,2}, ..., \boldsymbol{\Phi}^{T} \boldsymbol{x}_{k,C}]$$

$$= [\tilde{\boldsymbol{x}}_{k,1}, \tilde{\boldsymbol{x}}_{k,2}, ..., \tilde{\boldsymbol{x}}_{k,C}],$$
(3)

which is a matrix containing the transform coefficients $\tilde{x}_{k,i}$ of each patch in the group G_{x_k} . In Fig. 3 (c), \tilde{G}_{x_k} also denotes the group after DCT transform and each square block $\tilde{x}_{k,i}$ consists of all the coefficients of the corresponding patch in Fig 3 (b). For the sake of brevity, we call the block containing the transform coefficients $\tilde{x}_{k,i}$ the DCT patch for short in later reference

Thereupon, the Laplacian distribution of the DCT patch \tilde{x}_k could be formulated as

$$\tilde{\boldsymbol{x}}_{k}(t) \propto Laplace(\boldsymbol{\mu}_{k}(t), \boldsymbol{\sigma}_{k}(t)),$$

where

$$\boldsymbol{\mu}_{k}(t) = \sum_{1 \le i \le C} \boldsymbol{w}_{k,i} \cdot \tilde{\boldsymbol{x}}_{k,i}(t), \qquad (4)$$
$$\boldsymbol{\sigma}_{k}^{2}(t) = \frac{1}{C} \sum_{1 \le i \le C} (\tilde{\boldsymbol{x}}_{k,i}(t) - \boldsymbol{\mu}_{k}(t))^{2}.$$

In Eq. (4), $\boldsymbol{w}_{k,i}$ represents the weight measuring the similarity between $\tilde{\boldsymbol{x}}_{k,i}$ and $\tilde{\boldsymbol{x}}_k$, $\boldsymbol{\mu}_k(t)$ and $\boldsymbol{\sigma}_k^2(t)$ are the expectation and variance of the *t*-th coefficient in the DCT patch $\tilde{\boldsymbol{x}}_k$ respectively. Then, Eq. (4) is equivalent to the following expression

$$\boldsymbol{P}(\tilde{\boldsymbol{x}}_{k}(t)) = \frac{1}{\sqrt{2}\boldsymbol{\sigma}_{k}(t)} \exp(-\frac{\sqrt{2}}{\boldsymbol{\sigma}_{k}(t)} \big| \tilde{\boldsymbol{x}}_{k}(t) - \boldsymbol{\mu}_{k}(t) \big|).$$
(5)

Based on the assumption that the similar patches in one group are i.i.d., $\boldsymbol{\mu}_k$ and $\boldsymbol{\sigma}_k^2$ are shared by all the DCT patches $\tilde{\boldsymbol{x}}_{k,i}, (1 \le i \le C)$ in this group. So we get

$$P(\tilde{\boldsymbol{G}}_{\boldsymbol{x}_{k}}) = P(\tilde{\boldsymbol{x}}_{k,1}, \tilde{\boldsymbol{x}}_{k,2}, ..., \tilde{\boldsymbol{x}}_{k,C})$$

$$= \prod_{1 \le i \le C} \prod_{1 \le i \le S^{2}} P(\tilde{\boldsymbol{x}}_{k,i}(t))$$

$$\propto \exp(-\sum_{1 \le i \le C} \sum_{1 \le t \le S^{2}} \frac{1}{\boldsymbol{\sigma}_{k}(t)} |(\tilde{\boldsymbol{x}}_{k,i}(t) - \boldsymbol{\mu}_{k}(t))|)$$

$$= \exp(-\sum_{1 \le i \le C} \left\|\boldsymbol{\tau}_{k} \odot (\tilde{\boldsymbol{x}}_{k,i} - \boldsymbol{\mu}_{k})\right\|_{1}),$$
(6)

where S^2 is the patch size, $\tau_k = \sigma_k^{-1}$, and \odot stands for the element-wise product of two vectors. Hence, by maximizing probabilities $P(\tilde{G}_{x_k}), 1 \le k \le D$ of all the groups, the structured Laplacian sparsity prior for the entire image could be achieved as follows

$$\min_{\mathbf{x}} \sum_{1 \le k \le D} \sum_{1 \le i \le C} \left\| \boldsymbol{\tau}_{k} \odot (\tilde{\boldsymbol{x}}_{k,i} - \boldsymbol{\mu}_{k}) \right\|_{1},$$
(7)

where D is the total number of groups and C is the number of patches in each group.

Incorporating Eq. (7) as the regularization term into the optimization framework formulated by Eq. (2), we can rewrite it as

$$\min_{\mathbf{x}} \frac{1}{2} \| \mathbf{y} - A\mathbf{x} \|_{2}^{2} + \lambda \sum_{1 \le k \le D} \sum_{1 \le i \le C} \| \mathbf{\tau}_{k} \odot (\tilde{\mathbf{x}}_{k,i} - \boldsymbol{\mu}_{k}) \|_{1}.$$
(8)

3.3. Optimization Problem Solver Design

Having achieved the model to represent the Laplacian sparsity of the image group, the next step is to solve the optimization problem depicted by Eq. (8).

In this paper, we adopt the framework of split Bregman iteration to solve Eq. (8). The basic idea of SBI is to convert the unconstrained minimization problem into a constrained one by introducing the variable splitting technique and then invoke the Bregman iteration to solve the constrained minimization problem [19]. Numerical simulations show that it converges fast and only uses a small memory footprint, which makes it very attractive for large-scale problems [20].

First, by introducing a variable *z*, we can transform Eq. (8) into an equivalent constrained expression, i.e.,

$$\min_{\mathbf{x},\mathbf{z}} \frac{1}{2} \| \mathbf{y} - \mathbf{A}\mathbf{z} \|_{2}^{2} + \lambda \sum_{1 \le k \le D} \sum_{1 \le i \le C} \| \mathbf{\tau}_{k} \odot (\tilde{\mathbf{x}}_{k,i} - \boldsymbol{\mu}_{k}) \|_{1}, \text{ s.t. } \mathbf{z} = \mathbf{x}.$$
(9)

Then, applying Bregman algorithm [20] to Eq. (9) leads to the following three iterative steps:

$$\boldsymbol{z}^{(j+1)} = \arg\min_{z} \frac{1}{2} \| \boldsymbol{y} - \boldsymbol{A} \boldsymbol{z} \|_{2}^{2} + \frac{\eta}{2} \| \boldsymbol{z} - \boldsymbol{x}^{(j)} - \boldsymbol{b}^{(j)} \|_{2}^{2}, \quad (10)$$

$$\mathbf{x}^{(j+1)} = \arg\min_{\mathbf{x}} \frac{\eta}{2} \| \mathbf{z}^{(j+1)} - \mathbf{x} - \mathbf{b}^{(j)} \|_{2}^{2} + \lambda \sum_{1 \le k \le D} \sum_{1 \le i \le C} \| \mathbf{\tau}_{k} \odot (\tilde{\mathbf{x}}_{k,i} - \boldsymbol{\mu}_{k}) \|_{1},$$

$$\mathbf{b}^{(j+1)} = \mathbf{b}^{(j)} - (\mathbf{z}^{(j+1)} - \mathbf{x}^{(j+1)})$$
(12)

It is obvious to see that the optimization of Eq.
$$(9)$$
 is split

into two sub-problems, i.e., z and x sub-problems. Given x, the z sub-problem denoted by Eq. (10) is a minimization of a strictly convex quadratic function. Here, to avoid computing the matrix inverse, the steepest descent

avoid computing the matrix inverse, the steepest descent method is exploited to solve this equation by iteratively applying $\mathbf{r}^{(j+1)} = \mathbf{r}^{(j)} - \mathbf{c}^{(j)} = \mathbf{c}^{(j)}$ (12)

$$\boldsymbol{z}^{(j+1)} = \boldsymbol{z}^{(j)} - \rho \boldsymbol{d}^{(j)}, \qquad (13)$$

where ρ represents the optimal step, and $d^{(j)}$ is the gradient of Eq. (10) and calculated by

$$\boldsymbol{d}^{(j)} = \boldsymbol{A}^{T} (\boldsymbol{A} \boldsymbol{z}^{(j)} - \boldsymbol{y}) + \eta (\boldsymbol{z}^{(j)} - \boldsymbol{x}^{(j)} - \boldsymbol{b}^{(j)}).$$
(14)

Given z, the x sub-problem denoted by Eq. (11) is rewritten as

$$\min_{\boldsymbol{x}} \frac{1}{2} \left\| \boldsymbol{x} - \boldsymbol{r}^{(j)} \right\|_{2}^{2} + \frac{\lambda}{\eta} \sum_{1 \le k \le D} \sum_{1 \le i \le C} \left\| \boldsymbol{\tau}_{k} \odot (\tilde{\boldsymbol{x}}_{k,i} - \boldsymbol{\mu}_{k}) \right\|_{1}, \quad (15)$$

where $r^{(j)} = z^{(j+1)} - b^{(j)}$.

So now, the key to solving Eq. (8) is to find an efficient way to solve Eq. (15). To make it tractable, as in [12] we assume that each element of $\mathbf{x} - \mathbf{r}^{(j)}$ is i.i.d. with zero mean and the same variance. According to Theorem 1 proposed by [12], $\sum_{1 \le k \le D} \left\| \mathbf{G}_{\mathbf{x}_k} - \mathbf{G}_{\mathbf{r}_k^{(j)}} \right\|_F^2$ and $\left\| \mathbf{x} - \mathbf{r}^{(j)} \right\|_2^2$ satisfy the following equations with even the same variance have emphasible.

equation with a very large probability

$$\left\|\boldsymbol{x} - \boldsymbol{r}^{(j)}\right\|_{2}^{2} = \frac{N}{K} \sum_{1 \le k \le D} \left\|\boldsymbol{G}_{\boldsymbol{x}_{k}} - \boldsymbol{G}_{\boldsymbol{r}_{k}^{(j)}}\right\|_{F}^{2}, \quad (16)$$

where $K = D \times C \times S^2$.

Due to the orthogonality of DCT, the energies in the space domain and the frequency domain should be conserved. Then we have the equation below

$$\left\|\boldsymbol{G}_{\boldsymbol{x}_{k}}-\boldsymbol{G}_{\boldsymbol{r}_{k}^{(j)}}\right\|_{F}^{2}=\left\|\boldsymbol{\Phi}^{T}\boldsymbol{G}_{\boldsymbol{x}_{k}}-\boldsymbol{\Phi}^{T}\boldsymbol{G}_{\boldsymbol{r}_{k}^{(j)}}\right\|_{F}^{2}=\left\|\boldsymbol{\tilde{G}}_{\boldsymbol{x}_{k}}-\boldsymbol{\tilde{G}}_{\boldsymbol{r}_{k}^{(j)}}\right\|_{F}^{2}.$$
 (17)

Combining Eq. (16) and Eq. (17) with Eq. (15), we get $\min \frac{1}{N} \sum \sum \|\tilde{z} - \tilde{z}^{(j)}\|^2 + \frac{\lambda}{N} \sum \sum \|z - c(\tilde{z} - u)\|$

$$\max_{\mathbf{x}} \frac{1}{2K} \sum_{1 \le k \le D} \sum_{1 \le i \le C} \left\| \mathbf{x}_{k,i} - \tilde{\mathbf{r}}_{k,i} \right\|_{2} + \frac{1}{\eta} \sum_{1 \le k \le D} \sum_{1 \le i \le C} \left\| \mathbf{x}_{k,i} - \tilde{\mathbf{\mu}}_{k} \right\|_{1} \\
\min_{\mathbf{x}} \sum_{1 \le k \le D} \sum_{1 \le i \le C} \left(\frac{1}{2} \left\| \tilde{\mathbf{x}}_{k,i} - \tilde{\mathbf{r}}_{k,i}^{(j)} \right\|_{2}^{2} + \frac{\lambda K}{\eta N} \left\| \mathbf{\tau}_{k} \odot (\tilde{\mathbf{x}}_{k,i} - \boldsymbol{\mu}_{k}) \right\|_{1} \right),$$
(18)

which can be decomposed into $D \times C$ sub-problems as follows

$$\tilde{\boldsymbol{x}}_{k,i}^{(j+1)} = \arg\min_{\tilde{\boldsymbol{x}}_{k,i}} \frac{1}{2} \left\| \tilde{\boldsymbol{x}}_{k,i} - \tilde{\boldsymbol{r}}_{k,i}^{(j)} \right\|_{2}^{2} + \frac{\lambda K}{\eta N} \left\| \boldsymbol{\tau}_{k} \odot (\tilde{\boldsymbol{x}}_{k,i} - \boldsymbol{\mu}_{k}) \right\|_{1}.$$
 (19)

Obviously, Eq. (19) can be equivalently solved in an element-wise manner, i.e.,

$$\tilde{\mathbf{x}}_{k,i}^{(j+1)}(t) = \arg\min_{\tilde{\mathbf{x}}_{k,i}(t)} \frac{1}{2} (\tilde{\mathbf{x}}_{k,i}(t) - \tilde{\mathbf{r}}_{k,i}^{(j)}(t))^2 + \boldsymbol{\theta}_k(t) | \tilde{\mathbf{x}}_{k,i}(t) - \boldsymbol{\mu}_k(t) |,$$
(20)

where $1 \le t \le s^2$ and $\boldsymbol{\theta}_k(t) = \lambda K \boldsymbol{\tau}_k(t) / \eta N$.

By means of the soft thresholding algorithm of Lemma 2 in [13], we can arrive at a closed-form solution for Eq. (20) and accordingly, Eq. (19) can be solved and the solution is stated below

$$\tilde{\boldsymbol{x}}_{k,i}^{^{(j+1)}} = soft(\boldsymbol{r}_{k,i}^{(j)} - \boldsymbol{\mu}_k, \boldsymbol{\theta}_k) + \boldsymbol{\mu}_k.$$
(21)

Therefore, the corresponding patch in the space domain is

$$\mathbf{x}_{k,i}^{(j+1)} = \mathbf{\Phi} \, \tilde{\mathbf{x}}_{k,i}^{(j+1)} \,. \tag{22}$$

This process is applied for all the patches in each group.

3.4. Multi-Hypothesis Prediction and Summary

The way we reconstruct the image with all the achieved groups of patches is voting for each image pixel based on the multi-hypothesis theory. The following equation tells the reconstruction of each pixel $x^{(j+1)}(x, y)$ with each prediction produced from any patch of any group that covers the pixel location (x, y)

$$\mathbf{x}^{(j+1)}(x,y) = \frac{1}{\sum_{1 \le l \le B(x,y)} \beta_{(l)}(x,y)} \sum_{1 \le l \le B(x,y)} \beta_{(l)}(x,y) \cdot \mathbf{x}_{(l)}^{(j+1)}(x,y).$$
(23)

Here, B(x, y) is the total number of possible predictions for the current pixel, which indicates how many patches have overlapped this position. l is an index for the current prediction and $\beta_{(l)}(x, y)$ denotes its weight, which is set to a constant value 1 in the current implementation.

In light of all derivations above, the complete description of image CS recovery using structured Laplacian sparsity in DCT domain and multi-hypothesis prediction (SLS-MH) is given below:

 Table 1.
 Image CS recovery using SLS-MH

Input: The observed measurement y, the measurement matrix A and parameter λ ;

Initialization: set initial estimate $x^{(0)}$; for Iteration number $j = 0, 1, 2, ..., Max_iter$ Get $z^{(j+1)}$ by iteratively computing Eq. (13); Construct structured groups $G_{r_k^{(j)}}, 1 \le k \le D$ by searching for similar patches in $r^{(j)}$; *for* Each group $G_{r_k^{(j)}}$ Calculate μ_k and σ_k in Eq. (4); Obtain $x_{k,i}^{(j+1)}$ by computing Eq. (22); *end for* Get $x^{(j+1)}$ through multi-hypothesis prediction for each image pixel as in Eq. (23); Update $b^{(j+1)}$ by computing Eq. (12); *end for*

Output: Final recovered image \hat{x} .

4. EXPERIMENTAL RESULTS

In this section, experimental results are presented to evaluate the performance of the proposed image CS recovery algorithm using SLS-MH. Five standard gray images are tested, which are 'Cameraman', 'Foreman', 'House', 'Lena' and 'Vessels'. In our experiments, the CS measurements are obtained by applying a Gaussian random projection matrix to the original image signal at block level, i.e., block-based CS with block size of 32×32 [17]. SLS-MH is compared with six representative CS recovery methods in literature, total variation (TV) method [7], CS recovery in traditional DCT domain (DCT for short), wavelet method (DWT), collaborative sparsity (CoS) method [13], multi-hypothesis (MH) method [11]and SGSR [12]. It is worth emphasizing that SGSR is known as the state-of-the-art algorithm for image CS recovery.

In our implementation, all the parameters of SLS-MH are set empirically for all test images. Concretely, the size of each patch S^2 is set to be 8×8, the size of training window for searching matched patches L^2 is set to be 20×20, the number of selected similar patches C = 10 for each group, and λ is 5.0e-4. It is necessary to stress that the choice for all the parameters can be generalized to other natural images, which has been verified in our experiments. In this paper, we exploit the results of MH as initialization of the proposed SLS-MH for image CS recovery. The PSNR comparisons for all the test images in the cases of 20% through 40% measurements are provided in Table 2. SLS-MH provides quite promising results, achieving the highest PSNR among the seven comparative algorithms over all the cases, which can improve 4.98 dB, 6.64 dB, 3.50 dB, 2.43 dB, 2.78 dB and 0.90 dB on average, compared with TV, DCT, DWT, CoS, MH and SGSR, respectively.

Some visual results of the recovered images by these algorithms are presented in Fig. 4 and Fig. 5. Obviously, TV, DCT and DWT generate the worst perceptual results. The CS recovered images by CoS and MH possess much better visual quality, but still suffer from some undesirable artifacts, such as ringing effects and lost details. Although, the most recent SGSR method provides relatively pleasant results, it over smoothens the images and loses a lot of detailed textures. Our proposed SLS-MH algorithm not only yields the highest objective score in PSNR, but also preserves the fine details in the images and shows much clearer and better visual results than the other competing methods. The high performance of SLS-MH is attributed to the structured Laplacian sparsity based on the non-local similarity, which offers a powerful mechanism of characterizing the structured sparsity of natural image signals. Also, the multi-hypothesis contributes to the final results. Seen from Table 2, SLS represents our proposed algorithm without the MH predictions. It beats all the other methods except for SGSR, and can be improved a lot when MH is utilized.

The complexity of SLS-MH is provided as follows. Assume that the number of image pixels is N, and that the average time to search similar patches for each reference patch is T_s . The DCT operation of each patch x_k with size of S^2 is $O(S^2\log(S))$. Hence, the total complexity of SLS-MH

 Table 2.
 PSNR comparisons with different image CS recovery methods (dB)

Ratio	Images\Algorithms	TV	DCT	DWT	RCoS	MH	SGSR	SLS	SLS-MH
20%	Cameraman	25.29	23.95	25.70	27.78	26.05	26.66	26.47	27.19
	Foreman	32.31	29.36	33.88	34.29	34.61	36.08	36.17	37.18
	House	31.51	29.63	33.11	33.33	33.87	35.75	35.83	36.87
	Lena	30.65	30.47	32.73	32.48	33.08	34.05	33.26	34.12
	Vessels	22.08	20.38	24.98	25.98	26.09	30.61	27.12	31.31
	Average	28.37	26.76	30.08	30.77	30.74	32.63	31.77	33.33
30%	Cameraman	27.61	26.06	27.89	29.87	28.03	28.64	28.57	29.16
	Foreman	35.06	33.01	36.14	36.05	36.34	37.86	38.30	39.26
	House	33.64	31.99	35.25	35.57	35.81	37.51	37.65	38.70
	Lena	32.53	32.50	34.71	34.17	35.04	35.98	35.03	36.10
	Vessels	24.97	23.18	27.49	30.98	29.90	34.53	30.58	35.61
	Average	30.76	29.35	32.29	33.33	33.02	34.90	34.03	35.76
40%	Cameraman	29.86	27.95	29.87	31.40	30.05	30.48	30.49	31.09
	Foreman	37.33	33.22	38.16	38.64	38.16	39.45	40.19	41.24
	House	35.52	33.94	36.70	37.59	37.24	39.06	39.24	40.30
	Lena	34.16	34.19	36.14	36.16	36.22	37.58	36.53	37.68
	Vessels	27.77	25.68	29.87	34.23	32.93	37.36	33.54	39.26
	Average	32.93	31.00	34.15	35.61	34.92	36.78	36.00	37.91

is $O(N(S^2\log(S) + T_s))$. For a 256×256 image, the proposed CS recovery algorithm requires about 4~5 minutes for on an Intel Core2 Duo 2.96G PC under Matlab R2011a environment.



(e) CoS (34.29 dB) (f) MH (34.61 dB) (g) SRSG (36.08 dB) (h) SLS-MH (37.18 dB)
 Fig. 4. Visual comparison of CS recovered results for Foreman by different methods (ratio = 20%) and relevant PSNR values are shown in brackets.



(e) CoS (25.98 dB) (f) MH (26.09 dB) (g) SGSR (30.61 dB) (h) SLS-MH (31.31 dB)

Fig. 5. Visual comparison of CS recovered results for Vessels by different methods (ratio = 20%) and relevant PSNR values are shown in brackets.

5. CONCLUSIONS

In this paper, we design an algorithm for image CS recovery by the structured Laplacian sparsity in the DCT domain. The nonlocal similarity of natural images is exploited during structuring the image and superior sparsity could be achieved in this way. This prior information is incorporated into the CS framework and an ℓ_1 optimization problem is formulated. By modeling the sparse representation in overlapped groups instead of blocks and selecting multiple similar patches in each group, multiple predictions for each pixel could be achieved so that the MH theory is employed to recover the original image. Experimental results prove that the proposed algorithm can beat the other methods with very high objective gain as well as generating better visual pictures.

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